Waves - I

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Types of waves

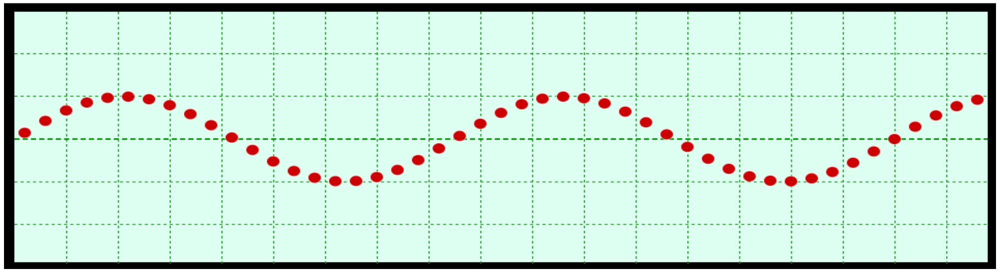
Wave equation & parameters

Speed of a progressive wave

Principle of superposition

Wave

A periodic disturbance that propagates itself in space is called a wave.



Types of waves

Based on movement of particles

- ❑ Longitudinal waves
- ❑ Transverse waves

Based on requirement of a medium

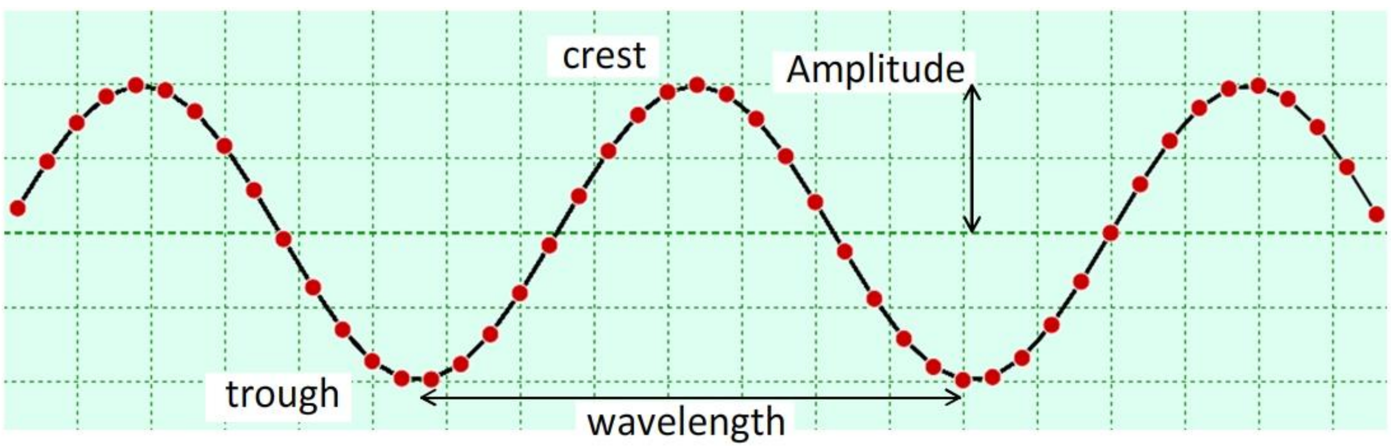
- ❑ Mechanical waves (require a medium for propagation)
- ❑ Non-mechanical waves (do not require a medium for propagation)

Based on propagation of energy

- ❑ Progressive waves (non-zero transfer of energy)
- ❑ Stationary waves (zero net transfer of energy)

Transverse waves

- These are the waves in which direction of oscillations of particles of the medium is perpendicular to the direction of propagation of the wave.
- Examples : waves in a stretched string, waves of the surface of water, electromagnetic waves (non-mechanical transverse waves)
- In transverse waves crests are the points of the positive amplitude and troughs are the points of the negative amplitude.
- The minimum distance between two points in phase is called wavelength (λ)
- Transverse wave equation is given by $y = A\,\sin(\,\omega t \pm kx \pm \phi\,)$



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direction of propagation of wave

Longitudinal waves

- These are the waves in which direction of oscillations of particles of the medium is parallel to the direction of propagation of the wave.
- Examples : sound waves
- In longitudinal waves compressions (points of the maximum pressure) and rarefactions (points of minimum pressure) are formed.
- The minimum distance between two points in phase is called wavelength (λ)
- Longitudinal wave equation is given by $X = A\sin(\omega t \pm kx \pm \phi)$

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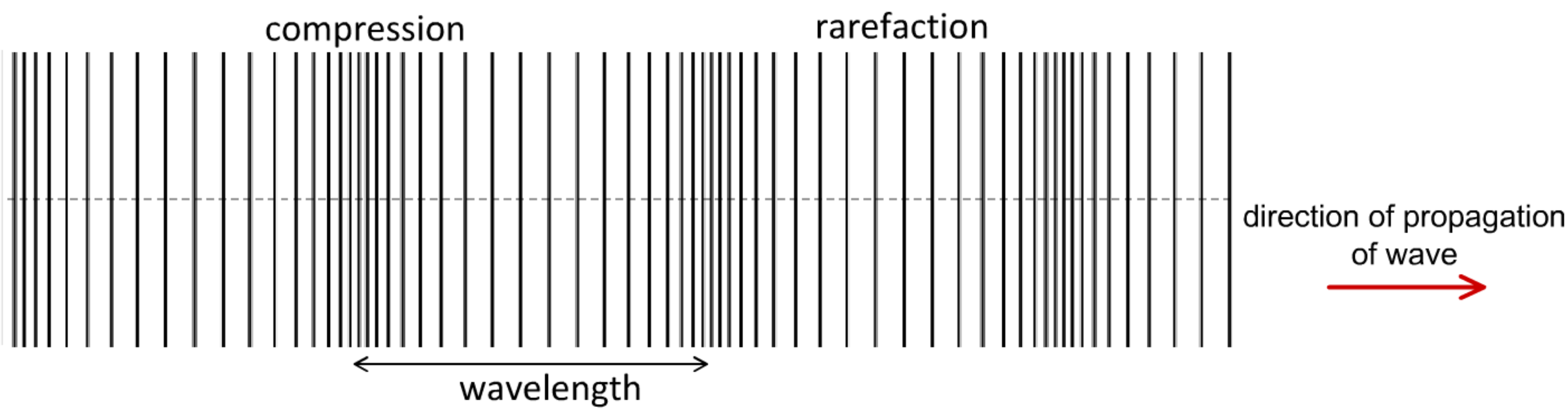


The wave equation

$$\boxed{y = A\sin(\,\omega t \pm kx \pm \phi\,)}$$

y	:	instantaneous position of a point/particle in a wave
A	:	amplitude of oscillation of parameter/particle in a wave
ω	:	angular frequency of oscillation of parameter/particle in a wave
t	:	instant of time
k	:	wave vector / propagation constant
x	:	position of a particular point in space containing the wave
ϕ	:	initial phase / phase constant / epoch

$$\omega = \frac{2\pi}{T} \qquad T = \frac{1}{n} \qquad k = \frac{2\pi}{\lambda}$$

$(\,\omega t + kx\,)$: for wave propagating along negative x-axis

$(\,\omega t - kx\,)$: for wave propagating along positive x-axis

$(\,\omega t \pm kx \pm \phi\,)$: instantaneous phase of a particular point

Propagation constant or wave constant (k)

General equation of a wave with zero initial phase is given by the relation

$$y = A\sin(\omega t \pm kx)$$

Initial wave pattern is obtained by considering $t = 0$. Therefore

$$y = A\sin(kx)$$

Sine function is periodic after every change in phase angle of 2π. Therefore position (y) of particles are same at points located at x , $2\pi x$, $4\pi x$, $6\pi x$ … $n2\pi x$

$$y = A\sin(kx + n2\pi)$$

$$y = A\sin k\left(x + \frac{n2\pi}{k}\right)$$

Least distance between any two points having same displacement (y) is obtained by taking $n = 1$. Therefore

$$y = A\sin k\left(x + \frac{2\pi}{k}\right)$$

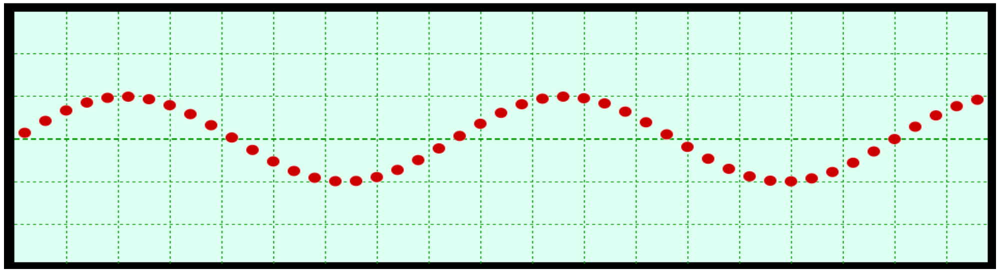
Wavelength (λ) is <u>defined</u> as the least distance between the particles that are in phase. Therefore

$$\lambda = \frac{2\pi}{k}$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

Velocity of a wave

When a wave propagates in space, shape of wave about a point (say a crest) remains constant during the wave propagation.



This implies that the phase of the wave equation remains constant

$$(\ \omega t - k x\ ) = \text{constant}$$

With passage of time, position (x) of a point with a particular phase propagates in the direction of wave while its phase remains constant.

Differentiating the above relation w.r.t. time we get

$$\omega \frac{\mathrm{d}}{\mathrm{d}t}(t) - k \frac{\mathrm{d}}{\mathrm{d}t}(x) = 0$$

$$k v = \omega$$

$$\boxed{v = \frac{\omega}{k}}$$

Using $\omega = 2\pi n$ and $k = 2\pi/\lambda$ we get

$$\boxed{v = n\lambda}$$

Velocity of waves (in different cases)

Velocity of a wave depends on the medium and the conditions in which the wave propagates.

- Velocity of wave is, in general, given by

$$\boxed{v = n\lambda = \frac{\omega}{k}}$$

n : frequency of the wave
λ: wavelength of the wave
ω : angular frequency
k : wave constant

- Velocity of transverse waves in a stretched string is given by

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

T : tension in the string
μ : linear density (mass per unit length) of the string

- Velocity of longitudinal waves in a medium is given by

$$\boxed{v = \sqrt{\frac{\gamma P}{d}}}$$

P : pressure of the medium
d : density of the medium
γ : ratio of specific heats of the medium